solucao do P1 7 de abril de 2009

Primeira questão:

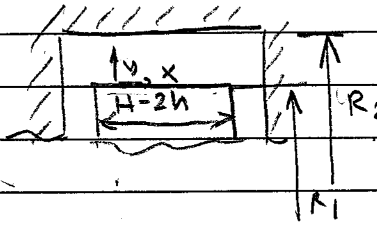
hipoteses: a) fluido newtoniano, incompressível

b) esc. laminar

c) folga radial e axial $\ll R$

d) perfil linear de velocidade nas folgas.

Torque devido à superfície lateral



$$T_1 = \int_A r\,dF = \int_A R_1 \tau_{yx}\,dA =$$
$$= R_1 \tau_{yx} 2\pi R_1 (H-2h) =$$
$$= 2\pi R_1^2 (H-2h)\,\mu \frac{\omega R_1}{R_2-R_1}$$

$$\therefore \quad T_1 = \frac{2\pi\mu R_1^3 (H-2h)\,\omega}{(R_2-R_1)}$$

Torque nos dois discos:

$$T_2 = \int 2\,r\,dF = \int_A 2\,r\,\tau\,dA = 2\int_0^{R_1} r\mu \frac{\omega r}{h} 2\pi r\,dr =$$

$$T_2 = \frac{4\pi\mu\omega}{h}\int_0^{R_1} r^3\,dr = \frac{4\pi\mu\omega R_1^4}{4h} = \frac{\pi\mu\omega R_1^4}{h}$$

$$T_{TOTAL} = \frac{2\pi\mu\omega R_1^3 (H-2h)}{(R_2-R_1)} + \frac{\pi\mu\omega R_1^4}{h}$$

a) $$T_{TOTAL} = \pi\mu\omega R_1^3 \left[\frac{2(H-2h)}{(R_2-R_1)} + \frac{R_1}{h} \right]$$

b) $\omega = 120\ RPM = \frac{(120)(2\pi)}{(60)} \frac{rd}{s} = 4\pi\ rd/s$, $\mu = 10^{-2}\ Pa.s$

$H = 20\ mm$, $h = 1\ mm$, $R_1 = 100\ mm$, $R_2 = 99\ mm$

$$T_{TOTAL} = (\pi)(10^{-2})(4\pi)(100\times10^{-3})^3 \left\{ \frac{2[20-(2)(1)]}{(100-99)} + \frac{100}{1} \right]$$

$$T_{TOT} = 5{,}4\times10^{-2}\ N.m$$

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segunda questão

$$u = x(1+2t) \qquad v = y$$

a) eq. trajetória

$$u = \frac{dx}{dt} = x(1+2t) \therefore \frac{dx}{x} = (1+2t)\,dt$$

$$\therefore \ln x = t + t^2 + \ln C \therefore \ln \frac{x}{C} = t+t^2 \therefore \frac{x}{C} = e^{(t+t^2)}$$

$$\therefore x = C e^{(t+t^2)}$$

em $t=0$, $x = x_0$ $\therefore$ $C = x_0$ $\qquad x = x_0 e^{t+t^2}$

$$v = \frac{dy}{dt} = y \therefore \frac{dy}{y} = dt \therefore \ln y = t + \ln C_1$$

$$y = C_1 e^t$$

$t=0$, $y = y_0$ $\therefore$ $\qquad y = y_0 e^t$

eliminando t: $x = x_0 \exp\left[\ln \frac{y}{y_0} + \ln^2 \frac{y}{y_0}\right]$

b) eq. linha de corrente

$$\frac{dx}{u} = \frac{dy}{v} \therefore \frac{dx}{x(1+2t)} = \frac{dy}{y}$$

$$\ln x = (1+2t)\ln y + \ln C \therefore \ln \frac{x}{C} = \ln y^{(1+2t)}$$

$$x = C\, y^{(1+2t)}$$

linha de corrente passando por x_0, y_0 em um dado t

$$x_0 = C\, y_0^{(1+2t)} \qquad C = x_0\, y_0^{-(1+2t)}$$

então $\quad x = x_0 \left(\frac{y}{y_0}\right)^{(1+2t)} \qquad \frac{y}{y_0} = \left(\frac{x}{x_0}\right)^{1/1+2t}$

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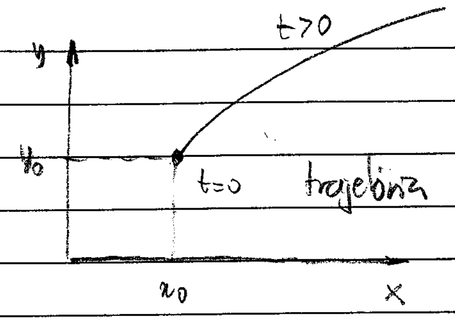

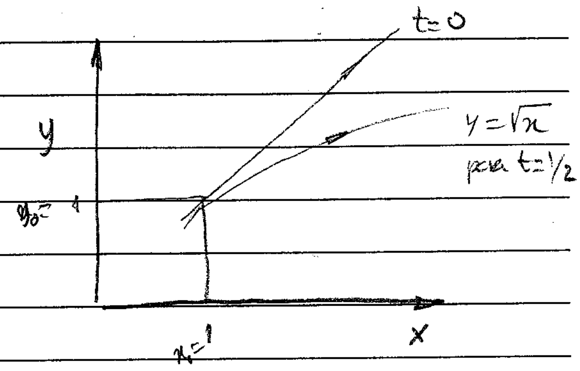

$$y = x^{\frac{1}{1+2t}} \qquad t = 1/2$$

$$y = x^{1/2}$$

3ª questão

na condição de tanque cheio a pressão no fundo é :

$$P_{\substack{fundo\\ cheio}} = \rho_g g H + P_{atm}$$

na condição com água :

$$P_{\substack{fundo\\ c/água}} = \rho_g g (H - h_1 - h_2) + \rho_{água} g h_1 + P_{atm}$$

igualando:

$$\rho_g g \cancel{H} + \cancel{P_{atm}} = \rho_g g (\cancel{H} - h_1 - h_2) + \rho_{água} g h_1 + \cancel{P_{atm}}$$

$$h_2 = \frac{(\rho_{H_2O} - \rho_g)\, h_1}{\rho_g} = \left(\frac{\rho_{H_2O}}{\rho_g} - 1\right) h_1$$

a) $h_2 = \left(\frac{\rho_{H_2O}}{\rho_g} - 1\right) h_1$

b) erro percentual

H = altura do tanque

$$V_{correto} = AH$$

$$V_{errado} = A(H - h_1 - h_2)$$

$$\text{erro} : \frac{V_{erro} - V_{correto}}{V_{correto}} = \frac{A(H - h_1 - h_2) - AH}{AH} = -\frac{(h_1 + h_2)}{H}$$

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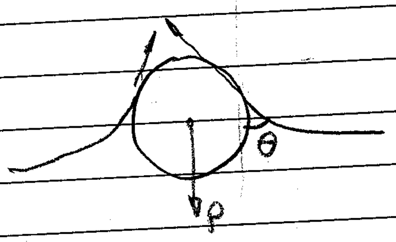
$$\varepsilon = \text{erro} = -\frac{(h_1 + h_2)}{H} \qquad \text{mas} \qquad h_2 = \overbrace{\left(\frac{\rho_{H_2O}}{\rho_g} - 1\right)}^{A} h_1$$

$$\varepsilon = -\frac{h_1(1 + A)}{H}$$

$$\varepsilon = -\frac{h_1}{H}\left(\frac{\rho_{H_2O}}{\rho_g}\right)$$

$$\varepsilon = -\frac{20}{300}\left(\frac{1000}{680}\right) \times 100 = -9{,}8\,\%$$

4ª questão



desprezando o empuxo (4 pelos)

$$P = (4)(2\sigma L \cos\theta)$$

$$P_{max} \rightarrow \theta = 0$$

a) $P_{max} = 8\sigma L$

considerando o empuxo:

4 pelos

$$P_{max} = 8\sigma L + \rho_L g \forall \qquad \forall = 4\frac{\pi D^2}{4} L$$

b) $P_{max} = 8\sigma L + \rho_L g 4\frac{\pi D^2}{4} L$

a) $P_{max} = (8)(72\times10^{-3})(5\times10^{-3}) = 2{,}88\times10^{-3}\ N = 0{,}29\,g$

b) $P_{max} = (8)(72\times10^{-3})(5\times10^{-3}) + (1000)(9{,}8)(\pi)(500\times10^{-6})^2(5\times10^{-3})$

$$P_{max} = 2{,}88\times10^{-3} + \underbrace{3{,}84\times10^{-5}}_{\text{empuxo é desprezível}}$$

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